ANALYTICAL METHODS FOR MARKOV PROCESSES

LORENZO DELLO SCHIAVO

ABSTRACT. The course (tentatively: 10 two-hour lectures) will be divided into two parts. The first part (7-8 lectures) will review the theory of Dirichlet forms and their relation to Markov processes.

A Dirichlet form is an *energy functional* (e.g., the Sobolev energy $\int |\nabla u|^2 \, \mathrm{d}x$) with a corresponding *generator* (e.g., the Laplacian $-\Delta$), and *semigroup* (e.g., the heat semigroup $t \mapsto e^{t\Delta}$). The semigroup is an integral operator that can be represented by a kernel $p_t(x, \mathrm{d}y)$, the *transition kernel* of a Markov process on a general state space (e.g., Brownian motion). We will show that this correspondence is essentially bijective, and that almost-sure properties of the Markov process may be deduced from properties of the corresponding form.

The second part (2-3 lectures) will explore in detail a single example.

REFERENCES. The first part of the course will follow standard textbooks [1, 2].

EXAM. The exam will consist of a seminar talk presenting proofs of selected results from the first part of the course, chosen in agreement with the lecturer.

PREREQUISITES. Participants are expected to be familiar with *basic* results in: real analysis (particularly, Sobolev spaces), probability (particularly, continuous-time Markov processes and Brownian motion), functional analysis (particularly, self-adjoint operators on Hilbert spaces), and measure theory.

FURTHER NOTES. The approach will be high-level and focused on the big picture: the aim is to introduce participants to the theory rather than to delve into technical details. The course will be held in either English or Italian, depending on the audience.

REFERENCES

[1] M. Fukushima, Y. Oshima, and M. Takeda. *Dirichlet forms and symmetric Markov processes*, volume 19 of *De Gruyter Studies in Mathematics*. de Gruyter, 2011.

[2] Z.-M. Ma and M. Röckner. *Introduction to the Theory of (Non-Symmetric) Dirichlet Forms*. Graduate Studies in Mathematics. Springer, 1992.

UNIVERSITÀ DEGLI STUDI DI ROMA "TOR VERGATA"
Email address: delloschiavo@mat.uniroma2.it